Theoretical and Experimental Jet Mixing of an Eccentric Primary Jet in a Constant Area Duct

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Theme

THE problem of turbulent mixing between two incompressible streams in a constant area duct with an eccentric primary flow jet is analyzed. A theoretical analysis is presented for the velocity profiles. An experimental investigation was performed for an eccentricity ratio (e/R) of 0.25, area ratios of 3 and 7.16, and a velocity ratio ranging from 1.2 to 2.9.

The theoretical analysis shows good agreement with the experimental results, especially for the cases with high area ratios and with low velocity magnitudes.

Contents

In many industrial applications it is necessary to deal with a jet expanding eccentrically into a confined stream of fluid. Typical examples are: jet engines, by-pass ducts, arrangements for reduction of aerodynamic noise level and many others.

The theoretical analysis of confined coaxial jet mixing is presented by Tabakoff and Khanna.[1] In their analysis they assumed that velocity profiles in the main region are similar to those of a freejet for which the universal function of the nondimensional excess velocity is

$$\Delta U/\Delta U_m = (1 - \xi^{1.5})^2 \tag{1}$$

where $\xi = Y/r$; r = freejet radius; Y = distance from the axis in the transverse direction; ΔU = excess velocity at Y; and ΔU_m = excess velocity at axis.

In the case of coaxial mixing, the velocity profile is taken to be the portion of the nondimensional velocity profile which lies between $Y = R$ and $Y = -R$, where R is the radius of the duct. For eccentric mixing, the velocity profiles are assumed to be the eccentric portion of the freejet. These profiles are not symmetric with the mixing duct axis, however, they are symmetric with the primary flow axis. Figure 1 shows the physical model with the corresponding notation.

In the case of a freejet, it was concluded that, owing to the similarity in velocity profiles, l/r will remain constant (l is the mixing length). Moreover, dr/dx = const = C or $r = Cx$. Now, $l = (r)$ (const) = (Cx)(const); from this last relation it is clear that the mixing length depends on C. Since the mixing length is the main parameter which specifies the degree of turbulence, the constant C is named the "turbulence constant." This constant was found to be equal to, 0.2 in the case of free jet mixing and 0.7 for confined jet mixing.[3] For confined eccentric mixing, the turbulence constant C is taken to be equal to 0.7.

Received January 12, 1972; synoptic received April 14, 1972. Full paper available from National Technical Information Service, Springfield, Va., 22151, as N 71-10010 at the standard price (available upon request). This work was sponsored under Contract Number DAHC04-69C-0016, U.S. Army Research Office—Durham.

Index category: Jets, Wakes, and Viscid Flow Interactions.

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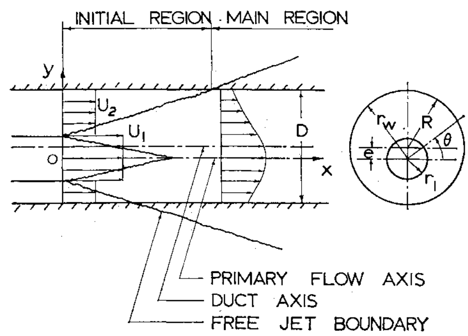

Fig. 1 Confined jet mixing with eccentricity.

The fundamental equations governing the flow of isoenergetic eccentric jet mixing are developed as follows. The continuity equation for one-dimensional flow for a control volume beginning at an arbitrary cross section at a distance x, and ending at the section where a uniform flow is attained, may be written as follows:

$$\int_0^A \rho U dA = \rho_3 U_3 A_3 \tag{2}$$

where the subscript, 3, refers to the uniform flow properties after complete mixing.

In order to integrate Eq. (2), using the universal excess velocity profile for turbulent freejets, as shown by Abramovich,[2] one subtracts the quantity

$$\rho_3 U_2 A_3 = \int_0^A \rho U_2 dA$$

from both sides of Eq. (2), and using polar coordinates the following equation is obtained

$$\Delta U_3 \rho_3 \pi R^2 = \int_0^{2\pi} \int_0^{y_w} \rho \Delta U y dy d\theta \tag{3}$$

where y_w is the wall radius measured from the primary flow axis.

Dividing each side of Eq. (3) by the quantity $\Delta U_m \pi R^2$, introducing $\xi = y/r$; $\xi_k = R/r$; $\xi_w = y_w/r$, and substituting Eq. (1) into Eq. (3), for the incompressible case, the following expression is obtained:

$$\frac{\Delta U_3}{\Delta U_m} = \frac{1}{\pi \xi_k^2} \int_0^{2\pi} \left(\frac{\xi_w^2}{2} + \frac{\xi_w^5}{5} - \frac{2\xi_w^{3.5}}{3.5} \right) d\theta \tag{4}$$

Upon substitution of $\xi_w \simeq \xi_k (1 + \bar{e} \cos\theta)$, where $\bar{e} = e/R$ one obtains

$$\Delta U_3/\Delta U_m = A_1(\xi_k) + \bar{e}^2 B_1(\xi_k, \bar{e}) \tag{5}$$

where $B_1(\xi_k, \bar{e}) = 0.5 - 2.5\xi_k^{1.5} + \xi_k^3(2 + 0.75\bar{e}^2)$ and $A_1(\xi_k) = 1 + 0.4\xi_k^3 - 1.143\xi_k^{1.5}$.

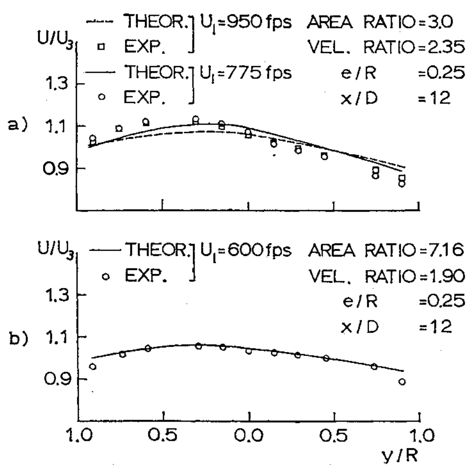

Fig. 2 Comparison between experimental and theoretical results.

As shown by Tabakoff and Khanna[1] the velocity distribution in the main region of the mixing duct, without any eccentricity of incoming jets, is given by

$$\Delta U_3/\Delta U_m = A_1(\xi_k)$$

which is the same expression given by Eq. (5) if one substitutes $\bar{e} = 0$. Thus, the second term in the expression for the velocity distribution can be thought of as a correction term due to the eccentricity. As can be seen, the second term is much smaller than the first one, owing to the fact that it contains $\bar{e}^2$ in the product, and for moderate values of $\bar{e}$, $\bar{e}^2$ is very small. Therefore, the velocity distribution in the main region of the mixing chamber is not very different for the two cases, i.e., with and without eccentricity, provided an appropriate axis of symmetry is selected.

Test Facilities: The test apparatus and the instrumentation used are the same which were used in the measurements of confined coaxially symmetric jets.[3] The only change made was the replacement of the centric primary flow pipes with eccentric ones.

The experimental data obtained correspond to two values of area ratio, 3.0 and 7.16, with an eccentricity of 0.5 in. A series of experimental data were obtained, as shown in detail in Ref. 4.

Comparison between the theoretical and experimental results in nondimensional form are shown in Fig. 2. Figure 2a shows the comparison, at a distance of twelve times the mixing duct diameter, for two cases with the same velocity ratio of 2.35, and with different velocity magnitudes. From this figure, it may be seen that the deviation between theoretical and experimental results increases as the velocity magnitude increases.

Figure 2b gives the comparison for the case with an area ratio of 7.16 and with velocity ratio 1.90. The theoretical calculations showed a better agreement with the experimental results as the X/D ratio was increased.

References

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[4] Tabakoff, W. and Hosny, W., "Theoretical and Experimental Jet Mixing of an Eccentric Primary Jet in a Constant Area Duct," Project Themis Rept. 70-11, AD 712333, July 1970, Univ. of Cincinnati, Cincinnati, Ohio.